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Design of optimal controllers for a three tank proces

S.Nagammai, S.Latha

Abstract

A typical three tank process has the difficulty in controller design because of assumed non linear flow and interaction between tanks. This paper deals with design methodology of full state feedback (FSFB) controller and linear quadratic controller (LQC) with pre compensator. The performance of proposed controllers is compared with Zeigler-Nichols (ZN) tuned proportional plus integral plus derivative (PID) controller for servo and regulatory response. The FSFB controller with pre compensator yields better performance compared to other controllers. The transient response specifications and performance indices are compared and which indicates the efficacy of FSFB controller over LQC. The stability analysis is carried out using the vector gain margin (VGM) index and it is evident that VGM lies well within the range for LQC than FSFB controller

Keywords: LQC, full state feedback controller, PID, State-Space model, Three tank process.

INTRODUCTION

The control of liquid level is a crucial problem in the process industries such as Petrochemical industries, paper making process or mixing process wherein series of tanks are used as processing unit. It is a challenging task for control system engineers to understand how the level control problem is solved. In order to understand the concepts of modern control theory the three tank process (TTP) is considered as an illustrative example. Design of State Feedback Controllers for a Nonlinear Interacting Tank Process and design of optimal Controllers for a Ball & Beam system is proposed by Nagammai et al [2,3]. A sliding mode control of MIMO system is proposed by Anouar Benamor et al [4] and which encounters complicated design and chattering of the manipulated variable. Fault detection and decision with Kalman filter applied to three tank process has been carried out by S.Abraham Lincon et al [5].

The disturbance rejection LQ control was demonstrated by Endre Borbély [6]. Ravi Kumar Jatoth et al proposed evolutionary algorithm based PID controller tuning for a TTP [7]. The mathematical modeling of three tank process is obtained from the mass balance equation expressed using Bernoulli's law. The Zeigler-Nichols (ZN) tuned PID controller is designed for comparative study.

The controllable companion form of state space modelling of the plant is obtained so as to design full state feedback controller (FSFB). A simulation of proposed controllers for three tank process is carried out using MatLab software.

State Space Modeling

The schematic shown in Fig.1 consists of three identical tanks coupled by an orifice. The input to tank 3 is $q_3(t)$ which is considered to be the disturbance variable to the system. Due to this external disturbance the level of tank 3 keeps varying. Hence the objective of the process is to control the level in tank 3 and to maintain at desired value by manipulating the inlet water flow $q_1(t)$. Hence it is considered as a single input - single output process (SISO).

Using the law of conservation of mass, the plant equations are expressed as given in equation (1).

The mass balance around tank 1 is,

$$\frac{dh_1}{dt} = f_1\left(h_1,h_2,h_3\right) = \frac{1}{A}\left[q_1 - q_{12}\right]$$

The mass balance around tank 2 is,

$$\frac{dh_2}{dt} = f_2\left(h_1,h_2,h_3\right) = \frac{1}{A}\left[q_{12} - q_{23}\right]$$

The mass balance around tank 3 is,

$$\frac{dh_3}{dt} = f_3\left(h_1,h_2,h_3\right) = \frac{1}{A}\left[q_{23} + q_3 - q_2\right] \tag{1}$$

Where

q_1 -- In flow rate to tank 1 (cm^3/sec)

q_2 -- Outflow rate of tank 3 (cm^3/sec)

q_{12} --flow between the tank1 & tank2 (cm^3/sec)

q_{23} --flow between the tank2 & tank3 (cm^3/sec)

q_3 -- Disturbance flow to tank 3 (cm^3/sec)

where

$$q_{12} = aC_1\sqrt{2g(h_1 - h_2)} \qquad with\ h_1 > h_2$$

$$q_{23} = aC_{12}\sqrt{2g(h_2 - h_3)} \qquad with\ h_2 > h_3$$

$$q_2 = aC_3\sqrt{2gh_3} \qquad with\ h_3 > 0$$

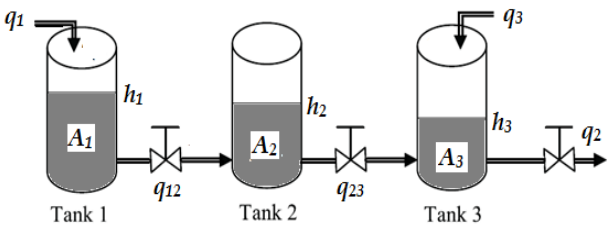

Figure 1: Schematic diagram of TTP

Table 1: Process variables and steady state values

Variable	Description	Value
g	*gravitational force*	*9.81 m^2/sec*
C_1	*Discharge coefficient of inlet orifice*	*1*
C_2	*Discharge coefficient of coupling orifice*	*0.8*
C_3	*Discharge coefficient of outlet orifice*	*1*
a	*Area of the connecting pipe*	5x10^{-5} *m^2*
A	*Area of each tank*	*0.0154 m^2*
h_{1S}	*Steady state water level of tank 1*	*0.5 m*
h_{2S}	*Steady state water level of tank 2*	*0.45 cm*
h_{3S}	*Steady state water level of tank 3*	*0.4 cm*
$q_{1S} = q_{2S}$	*Steady state flow rate*	*50 cm^3/sec*

The level in each tank is considered as state variable and the inflow to tank1 is considered as the input variable where as the disturbance variable is inflow to tank3.The level in tank 3 is considered as output variable. The state space representation of the three tank process in terms of state variables is obtained as follows:

$$\left[\dot{X}\right] = [A][X] + [B]u$$

$$Y = [C][X] + [D]u$$

The matrices A, B and D are,

$$A = \begin{bmatrix} \frac{-k_1}{A} & \frac{k_1}{A} & 0 \\ \frac{k_1}{A} & \frac{-(k_2+k_1)}{A} & \frac{k_2}{A} \\ 0 & \frac{k_2}{A} & \frac{-(k_2+k_3)}{A} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} \\ \frac{\partial f_2}{\partial q_1} \\ \frac{\partial f_3}{\partial q_1} \end{bmatrix} = \begin{pmatrix} \frac{1}{A} \\ 0 \\ 0 \end{pmatrix}$$

$$k_1 = \frac{aC_1\sqrt{2g}}{2\sqrt{\left(h_{1s} - h_{2s}\right)}}$$

$$k_2 = \frac{aC_2\sqrt{2g}}{2\sqrt{\left(h_{2s} - h_{3s}\right)}}$$

$$k_3 = \frac{aC_3\sqrt{2g}}{2\sqrt{\left(h_{3s}\right)}}$$

The output state equation is,

$$y = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

In this control design problem the Taylor's series expansion is used to linearise the plant model about the specified operating point and the state space model of the system is given in equation (2).

$$\begin{pmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \end{pmatrix} = \begin{bmatrix} -0.0322 & 0.0322 & 0 \\ 0.0322 & -0.0578 & 0.0257 \\ 0 & 0.0257 & -0.0371 \end{bmatrix}\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} + \begin{pmatrix} 64.93 \\ 0 \\ 0 \end{pmatrix}u \tag{2}$$

The system transfer function relating level in tank 3 to in flow to tank1 is,

$$\frac{H_3(S)}{Q_1(S)} = \frac{0.054}{S^3 + 0.1272S^2 + 0.0035S + 9.4\times10^{-6}} \tag{3}$$

Controllability property

The state variables of the third order system are defined as:

$$x_1(t) = h_3(t) = water\ level\ of\ tank\ 3$$

$$x_2(t) = \frac{dh_3(t)}{dt}$$

$$x_3(t) = \frac{d^2 h_3(t)}{dt^2}$$

The state input and state output of the third order system is defined as:

$$u(t) = q_1(t)$$

$$y(t) = h_3(t)$$

Let

$$\dot{x}_1(t) = \dot{h}_3(t) = x_2(t)$$

$$\dot{x}_2(t) = \ddot{h}_3(t) = x_3(t)$$

$$\dot{x}_3(t) = \dddot{h}_3(t)$$

From equation (3) it is obtained that,

$$\dot{x}_3(t) = -9.4\times 10^{-6}\, x_1(t) - 0.0035\, x_2(t) - 0.127\, x_3(t) + 0.054u(t)$$

Thus the state space representation of the three tank process in controllable canonical form is given in equation [4]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -9.4\times 10^{-6} & -0.0035 & -0.127 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad (4)$$

The controllability matrix is,

$$Q_c = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 64.94 & -2.6 & 0.17 \\ 0 & 2.1 & -0.2 \\ 0 & 0 & 0.05 \end{bmatrix}$$

The controllability property reveals that the system under consideration is controllable.

Design of PID Controller

The PID controller is able to provide an acceptable degree of error reduction along with stability and damping. In 1942 Zeigler Nichols proposed a tuning method using a frequency domain approach. A two point method is used in order to obtain the first order plus dead time model. In this method the time required for the response to reach 33.3% and 66.6% of the final steady state value is estimated. Using this data the delay time (t_d), process gain (k_p) and process time constant (τ_p) are determined as per the below mentioned formulae.

$$\tau_p = 1.43\left(t_{66.6\%\square y} - t_{33.3\%\square y}\right) = 328.6\sec$$

$$t_d = t_{33.3\%\square y} - 0.4\tau_p = 48.56\sec$$

$$K_p = \frac{\Delta y}{\Delta u} = 5744$$

The first order plus dead time (FOPDT) model thus obtained is given in equation (5)

$$G(s) = \frac{k_p e^{-t_d s}}{\tau_p S + 1} = \frac{5744 e^{-48.56S}}{328.6S + 1} \qquad (5)$$

The value of ultimate gain k_{cu} and period of sustained oscillation P_u is determined using magnitude and angle criterion. The values, thus obtained are given below:

$K_{cu} = 0.0019$ and $P_u = 0.034\sec$

The ZN-PID tuning parameters are,

$$K_c = 0.6 * k_{cu} = 0.0012$$

$$\tau_i = \frac{P_u}{2} = 0.017;\ K_i = \frac{K_c}{\tau_i} = 0.000013$$

$$\tau_d = \frac{P_u}{8} = 0.00425; K_d = K_c \tau_d = 0.028$$

Design of Full State Feedback Controller

In full-state feedback (FSFB) control technique all the state variables are fed- back to the input of the system through a suitable feedback gain matrix. In this approach, the desired location of the closed-loop Eigen values (poles) of the system is assumed to attain the desired transient behavior. As a rule of thumb the closed loop poles are moved ten times far away from the open-loop poles of the measured system so as to achieve the desired specifications. Hence, this approach is known as the state feedback controller design. The system must be a “completely state controllable” so as to design state feedback controller. Although the "optimum" location of the Eigen values of the closed loop system is guaranteed, the robustness to parameter variations and constraints is also ensured. Therefore, in order to compensate for offset, a pre compensator is added which eliminates the steady-state error in the response to the step input.

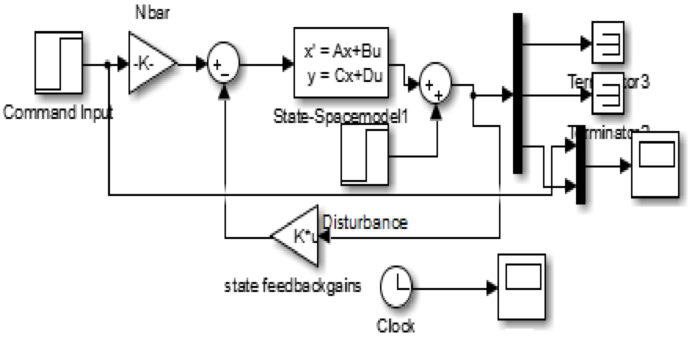

Figure 2: FSFB controller with pre compensator

The FSFB controller is composed of plant, state feedback gain matrix and a pre compensator. The primary need for adding pre compensator is to compute new reference input that increases the speed of system response, thus reduces the steady-state error to zero. The control law is given as $U = r\widehat{N} - KX$. $\widehat{N}$ is the gain of the pre compensator.

It is desired to have overshoot of less than 3% and settling time of at least 0.2 seconds, which corresponds to and $\omega_d = 25$ rad/sec. The closed loop poles p_1, p_2 and p_3 are thus chosen as $-25 \pm j25, -40$ .Now, the state feedback gains are obtained by solving the equation in (6)

$$det\left(\left[(SI - A) + BK\right]\right) = (s - p_1)(s - p_2)(s - p_3) \tag{6}$$

The closed loop response with FSBF controller results in large steady state error, and in order to compensate for this error a reference input compensation is included.

The state space model and output equation of the closed loop system with pre compensator is,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -90 & -101300 & -6\times10^7 \\ 0.03 & -0.058 & 0.026 \\ 0 & 0.026 & -0.04 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 69.94 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$$

The Simulink model of FSFB controller with pre compensator is shown in Fig.2. The state feedback gains for various pole locations are evaluated and listed in Table 2, which indicates that as the dominant poles are moved farther from imaginary axis, the speed of response increases.

Table 2: Full state feedback controller gains for various pole locations

Closed loop pole location	State feedback gains	Set point gain	VGM in dB
$-40 \& -25 \pm j25$	$[1.4 \quad 1560 \quad 923188]$	925410	8.5
$-40 \& -10 \pm j10$	$[0.92 \quad 478 \quad 147383]$	148070	11
$-40 \& -5 \pm j5$	$[1 \quad 214 \quad 36710]$	37016	12.5

Design of Linear Quadratic Controller

Linear Quadratic controller plays a vital role in many control design methods (Wilson 1996; Zadeh 1963; Ogata 2002). The theory of optimal control is concerned with operating a dynamic system at minimum cost. In linear quadratic control problem the system dynamics are described by a set of linear differential equations and the cost is described by a quadratic function. A linear Quadratic controller ensures better system stability than pole placement design.

The performance index (PI) is given by,

$$J = \int_0^{\infty} [X^T QX + u^T Ru]dt \tag{7}$$

Q is a positive semi-definite matrix which makes the scalar quantity $X^T QX$ to be always positive for all values of $X(t)$ and brings all states to equilibrium. R is a positive-definite matrix which makes the scalar quantity $u^T Qu$ to be always positive for all values of $U(t)$ that penalizes the control input. The objective is to select the optimal state feedback controller gains. The selection of matrix Q & R is the designer's choice. Depending on the choice of these matrices, the closed loop system will exhibit different set point tracking responses. Selecting 'Q' large, keeps 'J' small, so that the states are smaller. Small values of 'R' make the control effort less, so that the performance index 'J' given in equation (7) becomes small. Larger values of 'Q' and smaller values of 'R' results in location of closed loop poles far away from the origin which guarantees relative stability.

Consider a system described by the state space equation

$$\dot{X} = [A][X] + [B]u$$

$$y = [C][X] + [d]u$$

The optimal control, minimizing 'J' is given by the linear feedback law $U(t) = -K\,X(t)$

with $K = R^{-1}B^T P$,where 'P' is the unique positive definite solution to the Continuous Algebraic Ricattic Equation (CARE) given by $A^T P + PA + Q - PBR^{-1}B^T P = 0$

Let the scalar function be, $V(x) = X^T PX$ with $V(x) > 0$

The time derivative of $V(x)$ is, $\dot{V}(x) = \dot{X}^T PX + X^T P\dot{X}$

Now, $\dot{V}(x) = (AX + Bu)^T PX + X^T P(AX + Bu)$

$\dot{V}(x) = X^T (A^T P + PA)X + u^T B^T PX + X^T PBu$ From CARE we have, $A^T P + PA = -Q + PBR^{-1}B^T P$

Now,

$$\dot{V}(x) = (B^T PX + Ru)^T R^{-1}(B^T PX + Ru) - (X^T QX + u^T Ru)$$

Integrating $\dot{V}(x)$ we get

$$\int_0^{\infty} \dot{V}(x)dt = -J + \int_0^{\infty} (B^T PX + Ru)^T R^{-1} (B^T PX + Ru)dt$$

$$J = X^T(0)PX(0) + \int_0^{\infty} (B^T PX + Ru)^T R^{-1} (B^T PX + Ru)dt$$

The minimum value of 'J' is achieved when

$$U = -R^{-1} B^T PX = -KX$$

The design procedure is described by the following steps:

- ✓ The weighting matrices Q and R are selected.
- ✓ The Continuous Algebraic Ricatic Equation (CARE) is solved to get P matrix.
- ✓ The linear quadratic controller gain (K) is computed.
- ✓ The time response of the system is simulated.
- ✓ If the transient specifications are not met, then the weight matrices are tuned.

The value of Q is chosen as, $Q = diag[0, 0, 10000]$ and R is varied as R=0.1, 0.01, 0.001. As the objective is to control the level of tank 3, more magnitude of weight is given to state variable x_3. With this setting the linear quadratic controller gains and Eigenvalues are evaluated using the command

[K,P,E] = lqr(A,B,Q,R)and listed in Table.3 which indicates that small values of 'R' moves the Eigen values farther from imaginary axis, thereby increases the speed of response.

Table 3: Linear quadratic controller gains for various weight matrices.

R	LQR gains	Eigen Values	Set point gain	VGM in dB
0.1	$[0.077 \quad 6.15 \quad 307.3]$	$-2.6 \& -1.2 \pm j2.2$	316.2	25.95
0.01	$[0.11 \quad 13.4 \quad 980.7]$	$-3.8 \& -1.9 \pm j3.3$	1000	17.59
0.001	$[0.2 \quad 29.1 \quad 3120.5]$	$-5.5 \& -2.8 \pm j4.8$	3162.3	7.0

Stability Analysis

According to Franklin et al (2006), vector gain margin (VGM) is a single margin parameter that combines gain and phase margins into a single measure. This quantity eliminates the ambiguities that exist with the gain margin and phase margin combination in analyzing stability of a system. The original idea of VGM was proposed by Smith (1958) as cited in the work by Franklin et al (2006) is adopted in this work. The vector margin or disk margin is the distance measured from the Nyquist plot of the loop transfer function, including controller to the point (-1+j0) and the idea is illustrated graphically in Fig.3. Recent advances in computing facility have made measurement of VGM feasible. Due to difficulties in computing VGM, it was not being used in the past extensively.

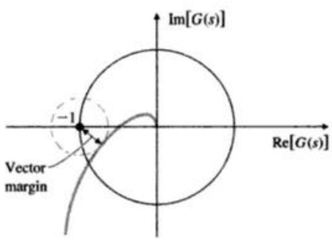

Figure 3: Stability analysis using VGM

VGM is linked with sensitivity function $S_{\infty} = \frac{1}{|1 + G(s)G_c(s)|}$ as the maximum of $\frac{1}{|1 + L(j\omega_{pc})|}$.

The reasonable values of VGM for good closed loop system stability are

$$3.5dB \le \max\left[\frac{1}{|1 + L(j\omega_{pc})|}\right] \le 9.5dB \qquad (6)$$

$L(j\omega_{pc})$ is the loop gain at the phase cross over frequency.

The VGM based stability assessment is carried out for the proposed control scheme and the results are tabulated in Table 2. The table shows that the FSFB controller with greater dominant pole value and linear quadratic controller with very small 'R' value yields VGM value which is well within the tolerance as specified in equation (6).

Stability analysis of PID controller

The closed loop transfer of the TTP with the controller setting as determined earlier is given by

$$\frac{H_3(S)}{H_{3sp}(S)} = \frac{0.0015S^2 + 6.5 \times 10^{-5} S + 7 \times 10^{-7}}{S^4 + 0.127S^3 + 0.005S^2 + 7.5 \times 10^{-5} S + 7 \times 10^{-7}}$$

In order to investigate the stability of the closed loop system the Bode diagram is drawn and is shown in Fig.10. It is clear from the plot that, though the system tracks the set point the internal stability is not ensured because GM & PM are not finite. The VGM is calculated as 88.3dB, which lies above the limit specified in equation (6).

Simulation Results

The state feedback gains and pre compensator gain are evaluated using MATLAB code. The open loop response of the designed system is shown in Fig. 4, and which indicates that, the open loop system is stable and non linear but set point tracking is not achievable. The transient response of the system with PID controller is shown in Fig.5. The servo response of the FSFB and LQ controller with and without pre compensator for various values of pole location is shown in Fig. 6 to Fig.9 for step change in set point. The servo regulatory response of the proposed controllers is shown in Fig.10. The magnitude plot of the closed loop system for various values of pole location is shown in Fig.11 & Fig.12. Further the magnitude plot indicates that the greater dominant pole value yields a lesser gain margin (GM). The lesser GM results in greater bandwidth that leads to increase in speed of response. The Nyquist plot of the closed loop system with proposed controller is presented in Fig.14. As all the contours corresponding to the loop transfer function $G(j\omega)G_c(j\omega)$ do not enclose the -1+j0 point, stability is assured.

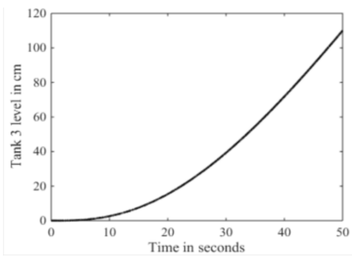

Figure 4: Open loop response of three tank process

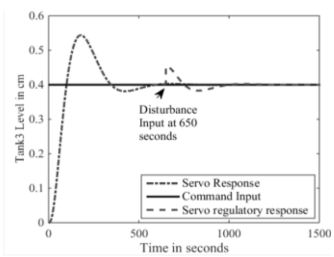

Figure 5: Servo regulatory response of PID controller

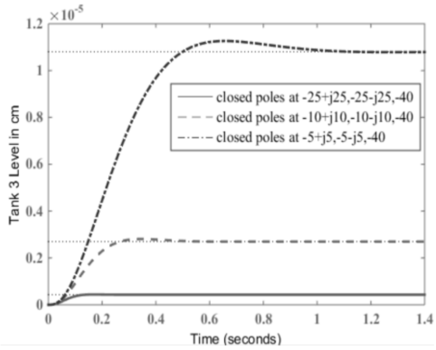

Figure 6: Servo response of FSFB controller

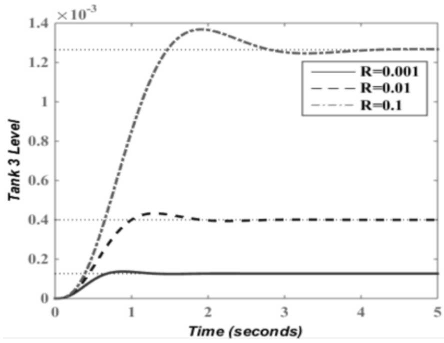

Figure 7: Servo response of linear quadratic Controller (LQC)

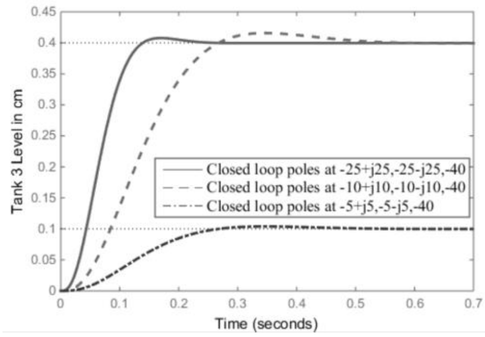

Figure 8: Servo response of FSFB controller with pre compensator

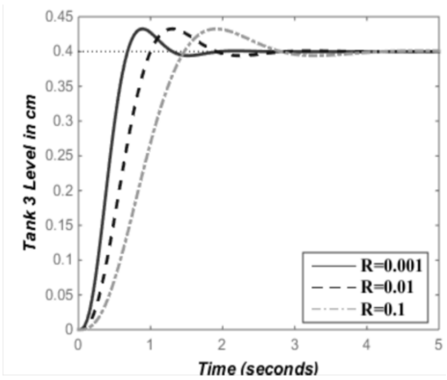

Figure 9: Servo response of LQC with compensator

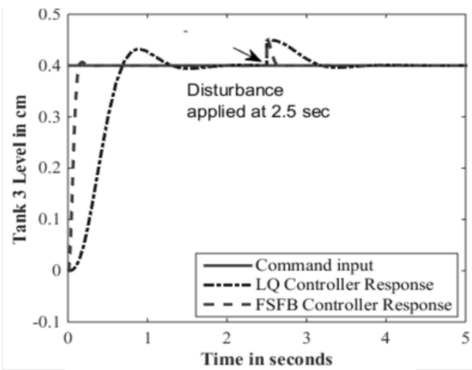

Figure 10: Servo regulatory response of proposed Controllers

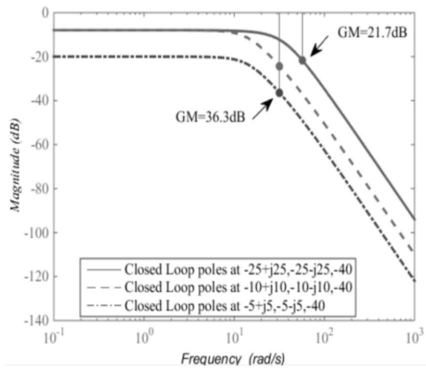

Figure 11: Bode magnitude plot of FSFB controller with pre compensator

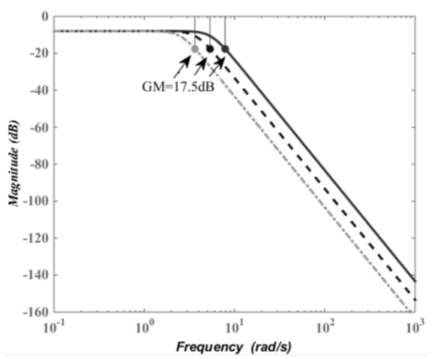

Figure 12: Bode plot of LQC with compensator

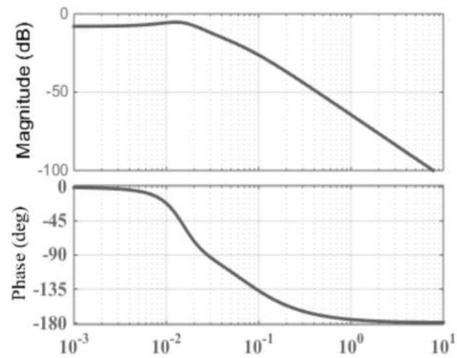

Figure 13: Bode plot of TTP with PID controller

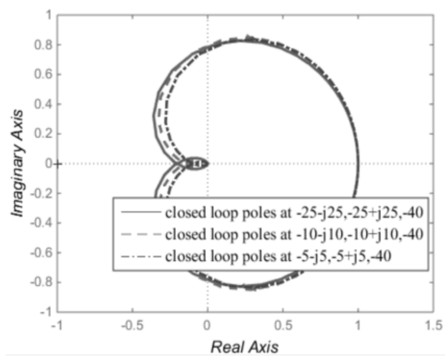

Figure 14: Nyquist plot of FSFB controller with pre compensator

CONCLUSION

The suggested algorithm is established for a three tank process. It has been shown that, the FSFB controller with pre compensator tracks the set point in desired settling time. Also, it exhibits less overshoot than PID controller. The performance summary is given in Table 3 which indicates that, the time domain specifications are close to the desired specifications with FSFB controller than with conventional other controllers. The execution with respect to settling time, peak overshoot, ISE, IAE are superior over PID controller. The disturbance rejection capability of FSFB controller is more beneficial than other controller, which is evident from Fig.8.

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